Web-based Assessment in Algebra and Calculus using MathXpert and WebGrades

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Abstract

We report on a new "homework management system" for algebra and calculus. The system is called *WebGrades*. It is designed to work with the existing software *MathXpert Calculus Assistant*, which is software to assist students in learning algebra, trigonometry, and calculus. The author used this system for four years (2010–2013) at San José State University.

1 Introduction

A Google search for "web-based assessment in calculus" will immediately reveal that this is a "hot topic." In 2010 there was a survey article on this subject in the *Notices of the American Mathematical Society* [7], but it is not our purpose to update that survey or write a new one. Instead, our purpose is to report on a new web-based assessment system, one which has been successfully used by the author for four years at San José State University, part of the California State University system, and to place such web-based assessment systems in a more general social and technological context.

1.1 Disintermediation

Web-based assessment can be considered as a special case of changes in commerce and society caused by the Internet. The term "disintermediation" is very much to the point. Disintermediation refers to "cutting out the middleman." For example, travel agents have been largely disintermediated by direct connections between travelers and providers of transportation, hotels, and cars. (Some have found related niches, so the species is not quite extinct.)

It is an oft-repeated thought that university professors are in danger of disintermediation, as purveyors of in-person, in-building lectures in traditional courses. We already have online courses delivered to thousands or tens of thousands of students, who may or may not be enrolled at a university.[1] This phenomenon seems destined to spread; some even assert

[1] This year, for the first time in its 150-year history, there is a course in which students at all the campuses of the University of California can enroll for credit: online calculus.

that the University itself is in danger of disintermediation.

As a general rule, the traditional way of doing something can be disintermediated by the Internet if, with the aid of the Internet, it can be done

- more cheaply
- more effectively
- more easily

If all three of those criteria are met, then disintermediation is a "done deal." The struggles begin when one or two of those criteria can be met, but not (yet) all three. For example, your doctor and hospital still keep your medical records in paper files, rather than electronic form. As an exercise, apply the above criteria to explain why these paper records (and the humans who maintain them) have not yet been disintermediated.

1.2 Disintermediation and the University

The traditional university class consists of three indispensable elements:

- Lectures
- Homework
- Tests

The disintermediation of lectures is projected to take place through the online availability of lectures, or shorter sequences of video instruction, and in addition, other aspects of what we may term "online instruction." This presently involves video of the "talking head", video of slides or blackboard work, and interactive computer graphics, usually presented using Javascript or applets. Although online instruction is not the focus of this paper, we still consider it in the light of the three criteria for disintermediation before moving on:

Does instruction meet the criteria for disintermediation?

- Online instruction is cheaper (per student), although there is an initial development cost.
- There are heated arguments about its effectiveness.
- It is easier for students and professors both, once the course is developed.

We will not discuss online instruction *per se* further in this paper; in particular we shall not go into any of the three bullet points above. Instead, we focus on the disintermediation of homework and tests. Collectively these two are known as "online assessment" or, since most such schemes today involve the Web, as "Web-based assessment."

An effective disintermediation of the traditional University course must simultaneously disintermediate all three elements: lectures, homework, and tests. We raise the following questions:

Basic questions about online assessment

- Can we implement online assessment across disciplines? That is, use the same framework for assessment in mathematics, physics, geology, history, and French?
- If not, what are the special needs of online assessment in mathematics?
- Is it necessary to integrate online assessment with online instruction?
- What are the significant differences between tests and homework? Can they be replaced by the same online tools?

As remarked above, there are significant development costs associated with online instruction. The same might prove to be true with online assessment. In the effort to make online instruction and assessment cheap, effective, and easy, some social and economic questions arise:

Social and economic questions

- Who should pay for the development?
- Who should pay for the use of the developed product and its "maintenance"?
- Who should have the right to use the developed product?
- What is an acceptable "pass rate"?
- Should we limit enrollment to those with sufficient preparation? and if so, what is "sufficient"?

No individual, and no single university, can answer these questions. I have both anecdotes and opinions. A few of the anecdotes are found below, but I have withheld most of the opinions and some of the anecdotes.

Closely related are the following technical questions, which we *can* answer (or at least, may be able to answer):

Feedbacks between assessment and learning

- How can we detect and remediate deficiencies in preparation?
- How can "learning outcomes" be certified?
- What motivates students, and what discourages them?
- What is the nature of the feedback loop between effective assessment and learning?

In contrast to the social and economic questions, we shall offer detailed answers to these questions.

1.3 The dismal ineffectiveness of traditional instruction

Here I want to establish that the bar to be met by the combination of online learning and online assessment is not very high. This requires facing some unpleasant facts, which are well-known but seldom discussed candidly. By "traditional instruction" I mean the process roughly described by the following plan for student activity:

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for each week in the course:
    learn the subject assigned for that week;
    get that learning assessed;
    go on to the next week;
```

Traditionally, "learning" meant attending the lectures and reading the text and doing the homework. Getting that learning assessed meant having the homework graded. It is also traditional to supplement that assessment with midterm exams and to "certify learning" with a final examination. The traditional method described here is not very effective, as every calculus instructor knows. While this point will be obvious to calculus instructors, this paper may have other readers, so let me detail the point with some anecdotal evidence. (These are not issues that attract carefully controlled research.)

First: Very few students actually read the textbook, and fewer than ten percent even look at the textbook before each lecture. I would always publish the lecture schedule and give credit for a three-minute quiz given at the beginning of each class. You could get half credit for this quiz if you just knew what the subject of the lecture would be, and full credit if you knew what Example 1 was about in the relevant section. It typically took several weeks of flunking these quizzes to train those students who attended lecture to look at the textbook before class. This was true in freshman calculus, but also in a course in differential equations for senior engineering students.

Second: Many students do not actually have the mathematical prerequisites. Calculus students often do not know their trig functions, cannot add algebraic or even numerical fractions, cannot perform more than the simplest algebra, and cannot perform arithmetic without their calculators. Senior engineering students cannot integrate by substitution or by parts, and many cannot even differentiate. At San José State, there is a placement examination for calculus, but the admission score is 70%, and there are no trigonometry problems on the test; so you can enroll in calculus knowing no trig and only 70% of the necessary algebra. I monitored errors on examinations in calculus to see whether they were errors in calculus, or errors in trig or algebra. They were about 80% errors in trig or algebra rather than calculus; and the problems were designed to *avoid* trig and difficult algebra as much as was consistent with the "intellectual integrity" of the course.

Third: The motivation of many students is only to somehow pass the course and be done with it. True, there are always some students who are enthusiastic. But two-thirds is considered normal lecture attendance! I checked that this is not just because of my bad lectures: other instructors have similar attendance, and I hear similar anecdotes from other universities. It is not at all uncommon for a student to miss ninety percent of the class meetings, turn in half the homework, earn fifty percent on the final, and expect to pass the course. (How that expectation was formed is beyond the scope of this paper.)

I once sent an email to 2500 people who had downloaded the free trial version of *MathXpert*, but not purchased the product, asking if they would be interested to purchase at half price, and asking for feedback about

why they had not purchased. I got hundreds of responses; these were surprisingly similar, and they said something like, "Awesome software! Thanks to *MathXpert*, I got an A, when I was about to flunk. But now that I passed my course, I'm *never* going to take another math course *ever again*, so I won't buy *MathXpert* at any price." These people were only motivated in the last month of their course, and only then by the fear of failure, so the then-available 30 day free trial was sufficient for their needs.

Fourth: The results on final exams are always disappointing, even to those with decades of teaching experience. Nobody goes out for a beer to celebrate their class's performance on calculus finals.[2] The "flunk rate" is always too high, even if the passing level is reduced to answering fifty percent of the questions correctly, and even if generous partial credit is given. I used to give out as "sample exams" the examinations from the past two or three years, some with solutions and some without. Whether that helped or not is not relevant here, except to the previous point about motivation.[3]

To summarize: the University has always accepted mediocrity in its mathematics courses, passing people, including engineers, with mediocre or worse mathematics skills. Hence, in order to disintermediate that system of traditional instruction in mathematics, online instruction and assessment only need to match that mediocre performance, and be cheaper.

1.4 Beyond Mediocrity

The word "assessment" almost implies a separation between two separate processes, learning and assessment. You first learn something, and then you get your learning assessed. Part of the reason for this separation of function is that, in traditional instruction, assessment bears a high cost:

Costs of traditional assessment

- Problems have to be made up, or selected from some source.
- Papers have to be collected.
- Papers have to be graded by a human being.
- There is a delay of at least days in getting the result of assessment.

Online assessment, at least the system to be described in this paper, reduces all but the first of these costs to zero:

Advantages of online assessment

- Homework submitted with one click (per problem)
- Instant feedback
- No grading cost (other than keeping a server up and running)

These efficiencies allow for the development of synergy between assessment and learning:

[2] Celebrating the end of finals week is another matter.

[3] It did, at least, largely prevent complaints about the exam containing unexpected or unfair questions.

Positive feedback from assessment to learning

- The system can help the student to solve the homework problems.
- Such help can remediate prerequisite deficiencies as well as help with the current topic.
- Resubmission can be allowed, motivating the student to learn if the first attempt is not successful.

The "active students" in my calculus classes at San José State (the ones who came to class and turned in homework) *all achieved one hundred percent on their homework.*[4]

2 MathXpert

All the discussion up to this point applies to any and all systems of online assessment and instruction in mathematics. The system to be described in this paper relies on software called *MathXpert*. This comes in three versions, *MathXpert Calculus Assistant*, *MathXpert Algebra Assistant*, and *MathXpert Precalculus Assistant*. The word "assistant" here is meant to emphasize that the student does the work, but the system can serve as advisor, giving hints or even showing the student what to do next. The student carries out a step-by-step solution, by selecting part of the current expression to work on, and then choosing a mathematical operation to apply. Figure 1 illustrates such a step-by-step solution, but does not show how the steps were taken. Figure 2 shows how a step is taken. The user has selected a part of the formula and then sees a menu of possible things to do with or to the selected term. These two figures also illustrate the upper and lower range of difficulty of problems that can be solved with *MathXpert*–one from advanced integration and one from the first days of algebra.

MathXpert has been described in several publications, including [1, 2, 3, 4, 5, 6], and therefore we shall not describe it in further detail here. Those interested can find further information at `www.HelpWithMath.com`, where they can also download a free trial version. This paper is devoted to an online assessment system called *WebGrades* that works with *MathXpert*. To understand how *WebGrades* works, however, it is important to understand one important point about *MathXpert*:

You can't make a mistake with MathXpert.

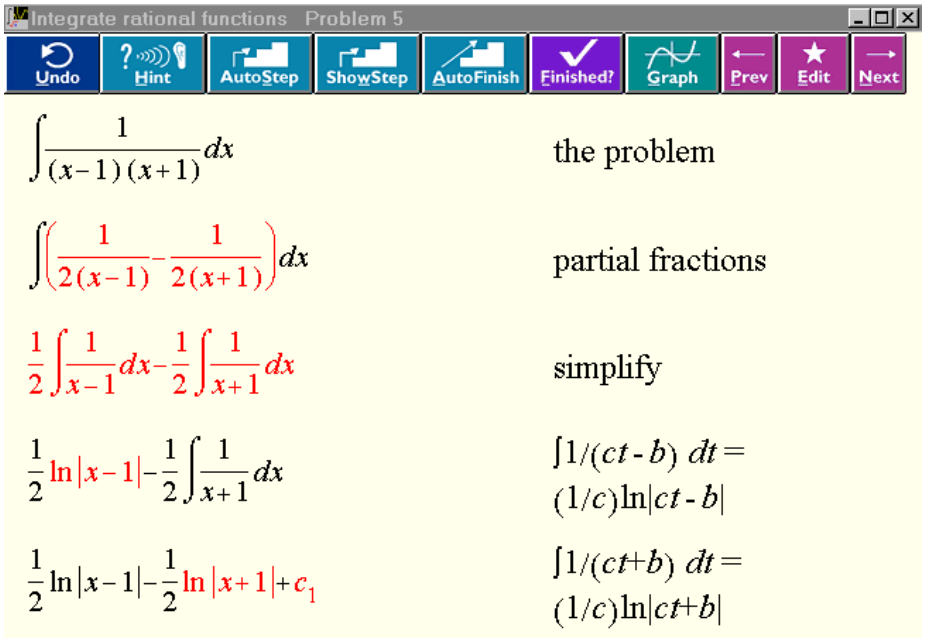
That is, it is impossible to generate an incorrect conclusion using *MathXpert*. For example, suppose you believe (incorrectly) that $\log(x + y) = \log x + \log y$. When you wish to apply that "law", you would select the formula $\log(x + y)$, but you would not find your incorrect "law" in the

[4]Many students chose to turn in their homework late. That was allowed, but you then had to work twice as many problems. It surprised me that many students would rather do twice as many problems than just do their homework on time. But this is one of the aspects of "easy" in the criteria for disintermediation. In traditional instruction, the student does not have this option.

menu of possible operations. You are then at a "teachable moment"; you can press the *Hint* or *Autostep* button for help.[5]

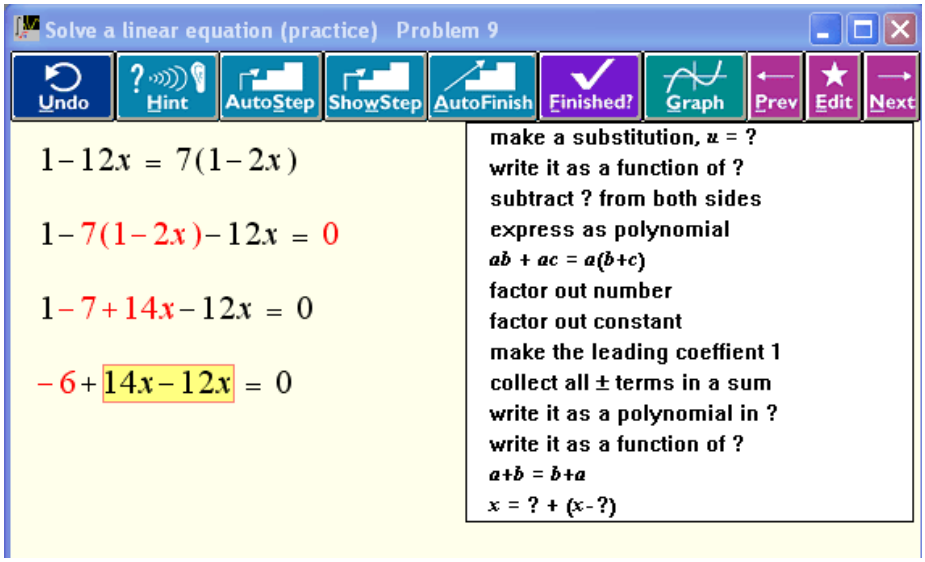
I once made a presentation of *MathXpert* to a group of high-school teachers. When they grasped the "no-mistakes" principle, one said with open-mouthed astonishment, "Why, that means everyone would get an A!" Her tone of voice made it clear that she found that unacceptable, but it is exactly what happened in the four years I used *MathXpert* teaching calculus. (Unfortunately they didn't also all get an A on the exams–that issue will be discussed in the last section of the paper.)

Figure 1: Step by step solution in *MathXpert*



[5] Unfortunately no software is psychic–it can't know what illegal step you *wanted* to take and provide you with the specific information that your step is not valid. However, in general it is pedagogically better to teach the correct way than the wrong way.

Figure 2: Taking the next step in *MathXpert*



3 WebGrades

Aside from general reflections about online assessment, the main point of this paper is to describe a particular online assessment system, *WebGrades*, that works with *MathXpert*. It can thus only be used in a course that requires the students to use *MathXpert* to do their homework; since *MathXpert* has, to put it mildly, not gone viral, *WebGrades* has also not been widely used; but there is another reason for that: this paper is the first public announcement of its existence.

How *WebGrades* works

- Students work homework using *MathXpert* (on their laptops).
- After solving each problem, they click *Submit as Homework*.
- The server keeps a record of which problems they solved and when.
- By the no-mistakes principle, wrong solutions are not possible, so the server only has to record a problem number and a date and match it to a student.
- By standard Web-and-database techniques, the up-to-the minute grades can be displayed in a table on the Web, for students and professors both to see.

Some questions immediately arise:

- What if the students had *MathXpert* take all the steps instead of doing it themselves?
- What about cheating by copying the work of a friend?
- How will the server recognize the student?
- How will the grades be presented on the web without violating privacy laws?

3.1 Can't I just use automode and do zero work?

I answered the first question this way: you must take at least half the steps yourself. This allows for the common scenario of making it past the difficult part of a problem, and then allowing *MathXpert* to take the last few steps (often just "cleanup") automatically (by pressing the *Auto Finish* button, or the *Auto Step* button repeatedly). This meant making a slight alteration to the *MathXpert* source code to keep track of which steps were taken automatically and which under student choice of operation. It still leaves the possibility of "cheating" by repeatedly using *Auto Step* or *Show Step* to see what to do, and then *Undo*, and then taking that step oneself. Actually, I demonstrated to the students how to do that, and said that's perfectly legal. If you can't solve a problem at first, you'll be able to after you solve it that way. I don't think many problems were solved this way, though, because students displayed genuine delight at getting problem after problem correct by themselves.

3.2 What about cheating?

We have already discussed the possibility of "copying" from MathXpert's auto mode by using *Undo*; I said this is not cheating. Suppose your roommate has worked problem 5 and prints out his solution and you have it in hand. Well, you could also have obtained a perhaps-even-better solution by pressing AutoFinish. To get credit for this solution, you will have to go through it step by step, taking each step yourself through the select-a-term and choose-the-operation method. This is also not cheating: you will learn by doing this. It won't work even if your roommate saves his solution as a *MathXpert* file and you open it, logging in using your email address, because when you open a file, the internal record of steps records that you did not take any of those steps, and you need to take at least half of them yourself to even see the *Submit* button. It also won't work if you try to submit his solution while he still has *MathXpert* open, at least if he is logged in under his own email address; because if he turns WebGrades off and you turn it back on (which is the only way to log out), again the number of steps you took will be zero. Of course, he could cheat for you by logging in as you, and taking all the steps himself. Short of requiring a thumbprint or other biological identification, there is no way to prevent that. After all, the possibilities for cheating on homework with *MathXpert* are considerably fewer than for cheating with paper homework.

3.3 Identifying the student on the server

Now we turn to the last two questions above, both of which involve legal issues. When I was a Caltech student in the 1960s, grades were posted by name on the professor's door. That would now be illegal. There is a national law in the United States, known as FERPA (Family Educational Rights and Privacy Act), which would be violated. It would allow posting grades by student ID number, but only with the student's written consent. But University policy (at SJSU) goes further: the student ID number is not private enough, so it cannot be used in public postings, and furthermore, it cannot be stored even temporarily on any off-campus server. Hence it is out of the question to identify students by their student ID number. Clearly the server needs a unique identifier for each student. For that purpose we simply use the student's email address. We ask each student to "register for WebGrades" at the beginning of the semester. This is done via a web form; the student gives his or her student ID number and email address. Since the student ID number is requested, this form has to be processed on a university server. But then the email address is used on an external server as a student identifier.[6]

3.4 Posting the grades

However, email addresses are also not private enough to use for posting grades. We therefore assign to each student a "secret number, known only to you and the professor." This number is automatically generated on the WebGrades server at the time of WebGrades registration, and is guaranteed unique. The student is informed of her secret number at the time of registration, and again (unobtrusively) every time she submits a homework problem. So, there is no problem with forgotten secret numbers: just submit another homework problem, or resubmit an old one (before the due date).

The WebGrades server displays the grades when the student clicks on *View the Grades* on the course web page. Clicking that link sends information identifying the course (but not the student) to the WebGrades server.

A typical display of grades is shown in Figure 3. The important points about the grade display are

Display of grades in WebGrades

- The display is always up to date (to the very second).
- The students see all the grades of their classmates, but identified only by secret number.
- The grades are displayed with one row per student, one column per assignment.

[6] Duplicates could theoretically arise if students fabricate email addresses that are not genuine. We explain the need for a unique identifier and promise not to send junk email, and this has not been a problem. The second registration for the same email address would be rejected.

Figure 3: Server-generated grade display by secret number and assignment

The following table shows the letter grades. The numbers shown are the standard University equivalents of letter grades: A = 4, B = 3, C = 2, D = 1, F = 0. When these are displayed in alphabetic form, the range 2.7 to 3.3 is a B, the range 3.31 to 3.49 is B+, 3.5 to 3.69 is A-, 3.7 to 4.0 is A, and similarly for B-, C+, etc. The highest possible grade is 4.25, which corresponds to 100 percent by the above formula. The lowest possible grade is 0.

The last column shows the average of the other grades in that row *for assignments whose due dates are today or in the past*. Thus problems you may have already submitted that are due in the future do not enter into this calculation. This is your homework grade up to and including today.

Student \ Assignment	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	20	21	22	Overall So Far
8320	4.25	4.25	4.25	4.25	4.25	4.24	4.25	4.25	4.25	4.18	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8343	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8366	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.18	4.18	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.24
8389	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8412	4.25	4.25	4.25	4.25	4.25	4.24	4.25	3.18	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.20
8435	4.25	4.25	4.25	4.25	4.25	4.24	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8458	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8481	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8504	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8527	4.25	4.25	4.25	4.25	4.25	4.24	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8550	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25
8573	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.25	3.00	3.63	4.25	4.16

3.5 WebGrades Summary

Basic elements of *WebGrades*

- Each student has *MathXpert* on their laptop.
- They connect to WebGrades from a menu item *Turn on WebGrades*.
- The first time they do that, they must enter their email address. After that it will be remembered.[7]
- They click *Submit as Homework* when they finish each problem.
- Server records the fact in a database along with the date, time, and student email and secret number.

4 Assignments in WebGrades

In this section, we address the issues related to problem selection, setting and changing assignments, and due dates. These issues can be considered with respect to online assessment systems in general, but in this section, we consider them in the narrow sense, and give the technical answers relevant to *WebGrades*. Many of these questions connect to wider economic and social issues that will be discussed in a later section.

4.1 Where do the problems come from?

I simply used the problems that come with *MathXpert*. There are about 6500 of them; typically 50 for each "topic". I would assign fewer than 25 of these, so that late submitters would still have the chance to solve twice as many and still get full credit. But *MathXpert* can read problems from a text file in a special format known as a "problem file", such as `Assignment12.hw`. The `.hw` is for "homework." So, in principle, a professor who is not happy with the built-in problems can prepare a custom

[7]Not on a lab computer. Originally *MathXpert* was also made available in a lab, but nobody used it. Every student has a laptop these days.

problem file, make it available on his or her course web page, and assign those problems for homework. I think that feature will rarely be used, as it requires slightly more effort for no obvious benefit. The issue of "where the problems come from" is a general one, not WebGrades-specific, and will be discussed in the concluding section.

4.2 How are the assignments and problems selected?

The instructor selects the assignments and their due dates. This information is stored in a text file on the WebGrades server. The file also, of course, contains information identifying the school, course, and instructor. Each assignment has a due date, and also a date for the last acceptable late submission. As I used WebGrades, the last-acceptable date was the date of the final exam, but it could be any date after the official due date. The rule that late submissions require twice as many problems is presently hard-coded, but that could easily be changed.

Part of the reason WebGrades has not been publicly announced is the lack of a clean Web-based interface to allow a new instructor to create this file. While that is not a difficult task, it has yet to be completed.

4.3 How are the assignments communicated to the students?

The students see a Web page, linked from the course web page (maintained by the instructor on the university's server). There is therefore a potential problem keeping this page in sync with the text file on the WebGrades server that stores the same information for use by the server. This problem would also need to be addressed before widespread deployment of WebGrades. For example, if the instructor wants to change the due date of Assignment 8 in the middle of the semester, or to push every due date three days into the future, both files need to be changed. This would argue for having the web page that the students see maintained automatically on the WebGrades server. But, the instructor may well wish to control this page, so as to be able to make remarks directly to the students, and have control of its formatting. Perhaps the best solution is to offer both options.

4.4 How and when does the server know what problem belongs in what assignment?

When a student submits a problem, a record is added to the database with the information that `JoeSmith@gmail.com` has solved problem 5 of topic 74 (or problem 5 from `customfile.hw`). At that time it is not linked to an assignment. The server confirms the receipt of the solution. The database also contains information that `JoeSmith@gmail.com` is enrolled in a certain class, identified by school, course number, instructor, and end

date.[8] But the fact that Joe has solved problem 5 of topic 64 is not linked to an assignment until the server is asked to display the grades for Joe's class. Then the server will look up the assignments for Joe's class, and will determine which of the assigned problems Joe (as well as the other students) have solved, and compute and display Joe's grade. The grade itself is not stored in the database, but is recomputed on each display.

4.5 What if a student works the wrong problems?

This can and does happen. People don't always read the assignment as carefully as they should. They get no feedback at the time of submission; nor should they, since they are allowed to work problems on past assignments (if late submissions are accepted) and they are also allowed to work problems on future assignments. Therefore, the students are encouraged to check after solving the first problem or two that their solution is showing up when they check their grades. Since the grade-reporting pages only show how many problems have been solved from each assignment, but not which problems specifically, I created another web page from which a student can enter their email or secret number, and see exactly which problems the server has recorded as submitted. This can be useful, for example, if they forgot where they left off in the middle of an assignment. This page is presently on the university's server, and is not maintained by WebGrades.

4.6 How are the grades determined, and how displayed?

There are two kinds of "grades" in WebGrades: percentage and letter. The percentage grade is just the fraction of the problems on an assignment that have been solved. If, for example, problems 1-25 on topic `integration by parts` have been assigned, but the topic actually contains 50 problems, then a percentage of 50% corresponds to a perfect score, i.e. 100% of the required problems. The letter grade is then computed from this score according to a rather strict formula, whose details are not important, except to note that this formula is not at present customizable to instructor preference. It is of little importance, however, since the active students always strive for perfection: everyone who hasn't given up on the whole course always gets 100%, even if they are late and have to work twice as many problems. So what counts for a C or B is not important, as nobody settles for a C or a B when an A is within reach and seems possible to them. Figure 3 (far above) shows part of a web page that the students can see by clicking on *View the Grades*. There is a different page to display the percentage grades. These pages are dynamically generated on the WebGrades server. The instructor has to place links to those pages on his or her course web page. Ideally, those links should be generated by the WebGrades server as part of an automated setup process when an instructor registers a new course with WebGrades;

[8] Note the importance of end date: a student can be enrolled in the course for a second time! and homework submitted last year should not count.

the instructor would then cut and paste them into the course web page. Presently they are sent to a new instructor by email.

4.7 Word problems

Algebra, trigonometry, and calculus traditionally contain two kinds of problems: symbolic problems (stated and solved in symbols), and word problems (stated in a "natural language", such as English or French. *MathXpert* can only assist students with symbolic problems and symbolic methods. Of course, you can use *MathXpert* for the symbolic part of the solution of a word problem, but it cannot assist in the translation from words to diagrams and symbols.

There are word problems in differential calculus, most notably in connection with "related rates." These typically involve two ships or airplanes on different courses, or ladders sliding down walls, or shadows. There are also word problems in connection with exponential decay, which in fact do not involve calculus at all in their solution. They occur in calculus books because calculus is used to derive the exponential decay equation that is actually used. For example, radioactive decay problems, or Newton's law of cooling. The problem sections of textbooks contain many more elaborate problems, designed to show that calculus is applicable in various fields of science, engineering, and medicine.

Therefore two of the assignments in the semester were word-problem assignments, solved and submitted on paper and graded by hand. Here, from the point of view of minimizing instructor's labor, WebGrades compares unfavorably with some other homework systems, which in some sense support word problems.

5 Discussion

In this section I offer some thoughts about online assessment and online instruction that are not tied to WebGrades, although they are informed by my experiences with WebGrades.

5.1 Tests *versus* homework

I take the view that *MathXpert* is a learning tool, but the ultimate aim should be that students can solve algebra and calculus problems without computer support, using pencil and paper. My students took traditional exams on paper in a classroom or lecture hall. (In fact, I don't even allow them to use calculators on exams, for which I was much villified.) To the extent that learning and assessment are separate activities, I viewed homework as part of "learning", and exams as "assessment."

That raises the question whether the use of *MathXpert* was actually successful in improving the symbolic manipulation skills of students. The use of *MathXpert* significantly improves the symbolic skills of students. This assertion is here made only as anecdotal evidence; scientific verification of this is discussed below. But anecdotally, I was very happy with the neat, clean, precise step-by-step solutions produced by some students on

exams. Of course, not all students got 100% even on the symbolic problems on exams; without *MathXpert* to catch their mistakes, they make mistakes and proceed. Sometimes they show ignorance of fundamental laws, and one wonders how they completed their *MathXpert* homework. They do not do perfect work on exams, but I think they do well enough on the symbolic parts. Their performance on word problems is not relevant here, as *MathXpert* and *WebGrades* were not used for those problems.

5.2 Attempts to quantify the effects of using online instruction or assessment

There was a study in the late nineties, soon after *MathXpert* was published in 1997, in a private high school in Finland. Four sections of calculus were involved; two were given access to *MathXpert* after school; in class, all four sections received the same instruction and homework. At the end of the semester, all sections took the same final exam. The sections without *MathXpert* performed as usual: many failures; there was a 12% and many other low grades on the final exam. In the sections with *MathXpert*, the lowest exam grade was 79%. No student got lower than a B (using the American equivalent grade). The study was planned to continue into a second semester, but it had to be stopped because the students in the control group demanded to be given access to *MathXpert*, and this demand was granted. (There is nothing more attractive than that which is forbidden. Maybe we should make calculus illegal.)

You will notice there is no citation for this study. The teachers who conducted the study attempted to publish their result in an education journal. The paper was rejected, because they had not arranged the control and experimental groups so that their average grades in previous math classes were equal. In fact they had no control over the assignment of students to sections; they just took the sections as the normal enrollment process produced them. But it turned out that the past year's math grades were slightly higher in the *MathXpert* sections, rendering the study "unscientific."

I had used *MathXpert* to teach with while developing it, but then until 2009 I was teaching computer science. When I began to use *MathXpert* at SJSU again in 2010, I tried to arrange a scientific comparison with another faculty member, whose students would not use *MathXpert*. Then all our classes would take a common final exam, and we could compare the results. Learning from the Finland debacle, we wanted to arrange that the control and experimental groups were comparable, by using the scores of the students on the Calculus Placement Exam, and discarding some students from the experiment in advance, thus forming well-matched experimental and control groups. But this proved impossible, because there are numerous ways to enroll in calculus without taking the placement exam (having AP calculus, having a sufficient score on certain standardized tests, etc.) Since students add and drop calculus in great numbers in the first weeks of the course, it is also not practical to administer a specially-designed pre-test to both classes.

Clearly there is nothing special about *MathXpert* in these stories. It is

profoundly difficult to conduct a scientifically acceptable demonstration that some educational method is better than another.

5.3 Textbooks and online homework

We are specifically talking about calculus textbooks here, but much of the discussion is more general.

- Calculus is 200 years old and hasn't changed significantly in the last 100 years.
- Some old textbooks are very good indeed.
- The leading textbook publishers put out new editions frequently and the new editions are very expensive, lengthy, and elaborate (e.g. with four-color graphics).
- With each new edition they are certain to revise the numbering of homework problems, so that old assignments won't work with the new edition, and every student must have the same edition.

This much is common knowledge. When I originally wrote *MathXpert*, I wanted to include the homework problems from common textbooks, so that *MathXpert* could be used easily on existing assignments. I discovered:

- Publishers consider the homework problems to be their principal intellectual property in a textbook.
- They think that if the homework problems were available elsewhere, students would not buy the textbook.

They are probably correct in that last thought, given that many students never read the textbook at all; especially in the Age of Google when students instinctively reach for the Web when they can't immediately solve a problem.

I told my students they could pay $4.50 for a used copy of any previous edition of the textbook, if they wanted, and could even buy a different textbook if they wanted, as all calculus books contain the same material. They saved over a hundred dollars each. The publisher's rep called on me in office hours and expressed her displeasure in no uncertain terms. At her urging I warned the students that next semester's professor might require them to purchase the current edition. Most still preferred to save $120.

While the current edition is still in use, the used-book market cuts into the new book market. Publishers try to combat that now, by "bundling" a homework management system with a textbook. The "bundle" includes a *single-user* license for limited-time use of the homework system. It is registered to the purchaser. The price of a standalone license is almost the same as the price of the bundle. The clear aim is to reduce the value of a used textbook; if you are required to have the homework system license, your rational choice is to buy the bundle. No wonder the publisher's rep came to my office!

5.4 A proposal

Encyclopedia Brittanica has been disintermediated by Wikipedia. Isn't it about time to disintermediate the textbook publishers? Why can't we have a Wiki textbook for calculus? Why can't we have Wiki textbooks for physics, biology, chemistry? The Kahn academy is working on disintermediation, but they are trying to disintermediate professors more than textbooks, and the approach is not Wiki-like.

Failing that: how about a Wiki database of homework problems? It could live on an AMS server; faculty the world over could assign problems from the AMS problem database. Any "homework management system" could use them, under licensing arrangements like those governing the use of Wikipedia. That would include, of course, pencil-and-paper "homework management." Separating the task of textbook writing from the task of creating homework problems would make both tasks easier. Textbooks old enough to be out of copyright probably contain many usable homework problems.

6 Appendix: Implementation details

WebGrades was implemented in about four days of work, using off-the-shelf software and standard Web-based programming techniques. This contrasts sharply with the effort that went into *MathXpert*: seven man-years over a twelve-year period. Specifically the components used were GSOAP, PHP, and MySql. PHP and MySql are standard tools for use in server-side programming. PHP generates "dynamic web pages" that display content fetched on demand from a database kept in MySql. A few short and simple PHP pages suffice to display the grades, add students to the database, etc. PHP is also used to write a "web service". Web services are another standard tool in server-side programming; they are used to permit a computer (program) anywhere on the internet to request services from a server. In WebGrades, a web service is used to respond to a remote student who has solved a problem, and record the problem number and student email in a database record.

At the other end of that connection is a student using *MathXpert*. When she clicks the *Submit as Homework* button, *MathXpert* needs to establish a connection to the WebGrades server, and then connect to the web service and send a the data about the solved problem. That is done using an off-the-shelf, open-source package called GSOAP, written in C. I was really quite astonished that within one day, I could download GSOAP, read its documentation, compile and link its source code to *MathXpert*, write a few lines of code to respond to the *Submit as Homework* button and call the web service written in PHP, and *it all worked*!

We have come a long way since the nineties, when off-the-shelf software often did not work. The standards for Web services are well-specified and well-understood; GSOAP is open-source software that has been worked on by a lot of people and evidently well-tested; PHP is used by thousands of programmers and the code that implements web services is also evidently well-tested. The code that I wrote must be a very small percentage of

the total code involved, most of which is not even visible to me. I knew about PHP and MySql, dynamic web pages and web services, having used them before and having taught them to computer science students; but I found out about GSOAP just by googling. That's life in the twenty-first century.

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