NEWTON REALIZES THE MOON IS FALLING

Newton thought about motion and its causes. He realized what Galileo taught: an object tends to stay in motion (moving in the same speed and direction) unless a force acts on it. Galileo had considered mainly motions in one direction. Newton noticed that it also requires a force to change the direction of a moving object. As far as I know, Newton was the first to realize that. Before Galileo, people thought that objects needed some kind of internal "force" to keep moving–left alone they would stop. After Galileo, they realized that an object moving in a straight line would keep moving unless something stopped it. But they seem to have not realized that if something that was *not* moving in a straight line (such as a planet or moon), it must be because something is pushing on it at right angles to the motion. That was Newton's first key idea, but it was still qualitative.

Applying this idea to the motions of the planets around the sun, he realized that there must be a force directed towards the sun (and *not* tangentially!). Similarly, the moon goes around the Earth, so there must be a force pulling it (or pushing it) towards the Earth. It was known that the moons of Jupiter revolved around Jupiter, so there must be an attraction between Jupiter and its moons, too. Thus the force that kept the planets in orbit was pointed towards the sun, not tangentially. Before Galileo, people (supposedly) thought that angels pushed the planets to keep them from stopping–those pushes had to be tangential. But Newton realized that the force had to be radial. So perhaps it had something to do with the sun. Newton knew Kepler's laws, and was a good enough mathematician to deduce from Kepler's laws that if there was a force it had to be radial, and it had to obey an inverse-square law. But what could that force be?

Sometime in 1666, the idea struck Newton: maybe that force that pulls the moon is the same force that pulls objects (such as the proverbial apple) to the ground. Indeed, maybe *everything* attracts *everything else*!

John Conduitt, Newton's assistant at the royal mint and husband of Newton's niece, had this to say about the event when he wrote about Newton's life:

> In the year 1666 he retired again from Cambridge ... to his mother in Lincolnshire and while he was musing in a garden it came into his thought that the power of gravity (which brought an apple from a tree to the ground) was not limited to a certain distance from earth, but that this power must extend much further than was usually thought. Why not as high as the Moon thought he to himself and that if so, that must influence her motion and perhaps retain her in her orbit, whereupon he fell a-calculating what would be the effect of that superposition.

Newton himself described his discovery as follows:

> From Kepler's rule of the periodical times of the Planets being in sesquialterate proportion of their distances from the center of their Orbs,I deduced

> that the forces which keep the Planets in theirOrbs must be reciprocally as the squares of their distances from the centers about which they revolves: and thereby compared the force required to keep the Moon in her Orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly. All this was in the plague years of 1665-66. For in those days I was in the prime of my age for invention . . . .

What was Newton calculating? He calculated the amount the Moon actually falls in one second (from the size of its orbit and the length of the month), and then calculated how far it should fall if it is falling just like an apple, but obeying an inverse-square law. Would the answers come out the same? Check it for yourself below.

Isaac Newton was 24 years old when he made that calculation.

Check the math yourself

The Moon doesn't get any closer to the Earth by falling, because the Moon is also moving sideways, and as it falls, the Earth's surface curves away beneath it. Let us calculate how far the moon falls in one second.

The moon takes about 29 days to go around the Earth once. To find out how many seconds that is, we multiply $60 \times 60 \times 24 \times 29 = 2505600$ seconds. The moon is about 30 Earth diameters away from Earth (as measured using solar eclipses). Let us take 8000 miles for the diameter of Earth; then how far does the moon travel per month? That would be $2\pi \times 240000 = 1507964$ miles. Dividing we find that the moon travels sideways

$$x = \frac{1507964}{2358720} = 0.64 \text{ miles} \;=\; 3380 \;\; \text{feet}$$

Now how far does it fall in one second? Let that distance be s. Let D be the diameter of the moon's orbit; so that is twice 240,000 miles, or 480,000 miles.

Figure 1 illustrates the situation, greatly exaggerating the distance the Moon moves in one second. By similar triangles we have

$$\frac{x}{s} = \frac{D - s}{x}.$$

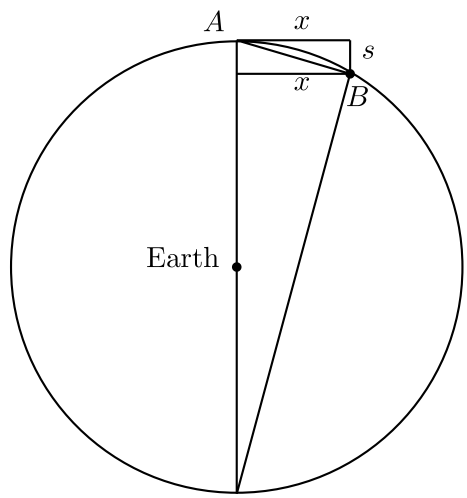
But s is negligible compared to D so we have almost exactly

$$\frac{x}{s} = \frac{D}{x}$$

which gives us $s = x^2/D$. That is

$$\begin{aligned} s &= \frac{x^2}{D} \\ &= x\frac{x}{D} = 3380\frac{0.64}{480000} \qquad \text{miles} \\ &= 0.0045 \;\; \text{feet} \\ &= 0.05 \text{ inches} \end{aligned}$$

FIGURE 1. The moon starts at A. In one second it moves to B, falling a distance s while moving horizontally a distance x. The diameter of the orbit is D so the vertical side of the big triangle is $D - s$.



One-twentieth of an inch. That's how far the moon falls per second.

Now how far would we expect the Moon to fall if it is acted upon by the same force that makes apples fall? The moon is 60 times as far from the center of the Earth as the apple; let us suppose the Earth attracts the apple as if all the mass was concentrated at the center. Then we would expect the force on the Moon to be smaller by a factor of 60^2, which is 3600. We know that the apple falls $\frac{1}{2}gt^2 = g/2$ feet on Earth; since $g = 32$ that is 16 feet, or 192 inches. if g goes down by a factor of 3600, the Moon should fall 0.053 inches in one second. One-twentieth of an inch. As Newton said, the two calculations "answer pretty nearly."

References

[1] Keesing, R. G., The History of Newton's apple tree, *Contemporary Physics* **39,** 377–91, 1998.