HIPPARCHUS MEASURES THE DISTANCE TO THE MOON

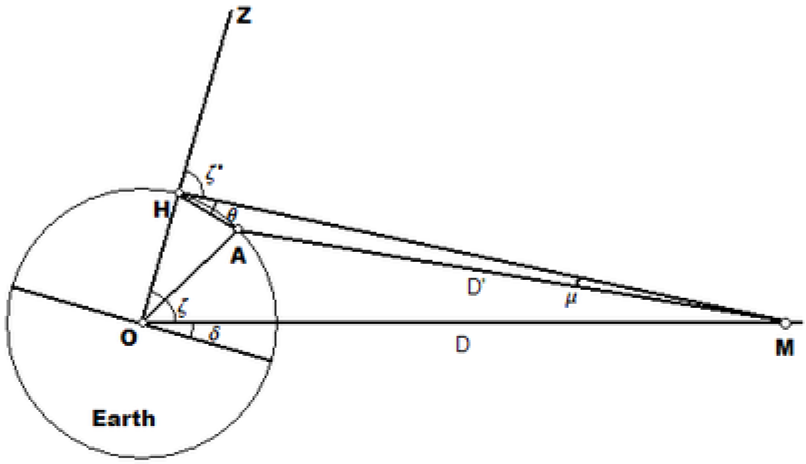
The date: March 14, 189 BCE. (Or maybe 190; sources disagree). The places: Alexandria and the Hellespont. The event: a solar eclipse. The Hellespont is a narrow strait on the northwest coast of Turkey. The eclipse was total on the coast of the Hellespont, but in Alexandria, one-fifth of the Sun had remained visible during the eclipse.

Here's a map showing these two locations:

The sun subtends an angle of about half a degree when viewed from Earth, and in saying that "one-fifth of the Sun remained visible" in Alexandria, apparently what was meant was that the visible part of the sun subtended an angle of about one-fifth what the whole Sun subtended, thus one-tenth of a degree.

In the diagram below, H is the Hellespont, A is Alexandria, and M is the moon. The two lines MH and MA show the path of light rays from the edge of the moon's shadow. Since the Moon takes a month to go around the Earth, we can assume it stays in one place during the short solar eclipse. Similarly, the Earth's yearly revolution around the Sun (or the Sun's revolution around the Earth, in the geocentric system) does not matter. The daily rotation (of the Earth or the Sun, it doesn't matter which for this purpose) makes the shadow of the Moon move across the Sun from west to east. Therefore only the difference in latitudes of A and H matter; it was not necessary to observe the eclipse at the exact same time in both places (luckily, since there were no accurate clocks). Those latitudes are 41° for H and 31° for A (and those were known to Hipparchus, at least within a degree). Since the difference is only ten degrees, the straight-line distance HA is approximately the same as $10/360$ times the circumference of the earth.

The sharp angle AMH is the "lunar parallax", which the eclipse showed was 0.1 degree. The diagram below shows the geometry of the situation.



In the diagram,

- δ is the declination of the Moon. That is how far above the plane of the equator the moon appeared that day; that was -3°, which would have meant that the moon rose to a maximum height of 3° less than the latitude. So δ is known if you watch the moon and know your latitude.
- ζ is the latitude of H minus δ.
- ζ' is a small fraction of a degree different from ζ, since OM and HM are almost parallel because the Moon is pretty far away.

Every angle in the diagram could be known before the eclipse, except the small angle $AMH = 0.1^\circ$, which was measured during the eclipse. That determines M and hence the distance D, in terms of the radius $OH = OA$ of Earth.

The angle AMH to be measured is the "lunar parallax" for the two locations A and H.

Hipparchus was perhaps the discoverer (or inventor?) of trigonometry. He didn't invent the sine and cosine functions, but instead he used the "chord" function, giving the length of the chord of the unit circle that subtends a given angle. He was able to solve the geometry problem to find D as a multiple of the Earth's radius, and concluded that the moon is 71 Earth radii away from Earth. In a fashion befitting a modern scientist, he reported his error bars: he said the moon was between 35 and 41 Earth diameters away. The modern value is about 60 radii, or 30 diameters.

Today you can measure the distance to the moon by bouncing a laser beam off a mirror left there by astronauts, and timing the round-trip of the light. We know the speed of light now very accurately, so to get the distance just requires a multiplication.

A few years later, Hipparchus made another attempt to measure the distance to the Moon, using a lunar eclipse, and he got a larger answer; and his error bars from the two calculations did not even overlap, which he honestly admitted. The measurement that the eclipse was 4/5 complete was probably not very accurate, modern calculations are said to show. We don't really know for certain the exact details of Hipparchus's calculations, though several historians have published reconstructions; only one fragment of his works has survived.

In principle, the accuracy of such measurements could be improved by using a longer baseline. That seems not to have been done until 1751, when the French astronomer Nicolas Louis de Lacaille at the Cape of Good Hope measured the lunar parallax by comparing his observations with observations made by other astronomers in Europe, and calculated that the lunar parallax that would be observed from opposite sides of the Earth (a baseline of 8000 miles) would be about two degrees.

References

[1] G. J. Toomer, Hipparchus on the distances of the sun and moon, *Archive for History of Exact Sciences* **14** (1974), 126-142.

[2] Hirschfeld, Alan W., *Parallax: The Race to Measure the Cosmos*, Freeman, New York (2001).